# CALCULATION OF THE SPATIAL STREAMLINE FLOW AROUND THE BLADES OF AN AXIAL TURBINE BY THE POTENTIAL FLUX OF AN INCOMPRESSIBLE LIQUID

V. P. Ryabchenko

UDC 532.5:621.22

During the past two decades considerable attention has been devoted to the flow of liquids or gases in axial compressors and turbines. In theoretical investigations an approximate solution of the problem of the spatial flow through a turbine was obtained by splitting the problem into two interconnected two-dimensional problems: calculating the average axisymmetrical flow through the blades of the apparatus and the flow through a grid situated on the surface of rotation in layers of varying thickness [1-3]. In a number of papers [4-11] the three-dimensional nature of the flow was taken into account approximately without this subdivision of the problem. The geometry of the blades was simulated either by a spatial array of plates [6-8] or by a spatial ring array of blades, the shape of which is parts of screw surfaces [10, 11]. The latter model obviously better describes blades of an axial turbine in general, but the method of solving the problem in [10, 11] is too complex for making commercial calculations and requires considerable computer time. In this paper we describe a more economic method of calculating the aerodynamic characteristics of a spatial ring array around which an incompressible liquid flows, which is based on the vortex theory or a propeller [12] and an impeller of finite swing [13].

1. We will consider the flow of a liquid through a single series of blades which rotate with constant angle of velocity $\omega$ in a coaxial cylindrical channel, which is infinite in the axial direction. The model of the flow is based on the following assumptions: 1) the incoming flow is a uniform flow with an axial velocity v at infinity, 2) the liquid is ideal and incompressible, and 3) the perturbations in the liquid are small compared with the unperturbed flow. We will introduce a Cartesian system of coordinates (x, y, z) and a cylindrical system of coordinates (x, $r^*$, $\theta^*$) connected with the rotating series of blades. The x axis is directed along the axis of the cylinders, and the y and z axes are taken in a plane perpendicular to it. The coordinates $r^*$ and $\theta^*$ are related to y and z by the usual equations: $y = r^*\cos\theta^*$, $z = r^*\sin\theta^*$, where the angle $\theta^*$, is measured in the positive direction from the y axis (Fig. 1).

We will assume that the blades $\Sigma_n$ ($n = 0, \ldots, N-1$) are infinitely thin, and their shape is close to the surfaces of the current of the unperturbed flow, which are parts of screw surfaces bounded in the ($r^*$, $\theta^*$) plane by the rectangle $\{r_1 \le r^* \le r_2,\ -\psi+\alpha_n \leqslant \theta^* \leqslant \psi+\alpha_n\}$. Here $\alpha_n = 2\pi n/N$, n is the number of a blade, N is the number of blades, $r_1$ is the radius of the inner cylinder, $r_2$ is the radius of the external cylinder, and $-\psi+\alpha_n$ and $\psi+\alpha_n$ are angles defining the position of the front and rear edges of the blades, respectively. Within the framework of the model considered the vortex trails which occur behind the blades due to a change in the circulation along the height of the blade will be assumed to be situated along the surfaces of the current $W_n$ of the unperturbed flow bounded in the ($r^*$, $\theta^*$) plane by the half-zone $\{r_1 \le r^* \le r_2,\ \psi+\alpha_n \leqslant \theta^* < \infty\}$.

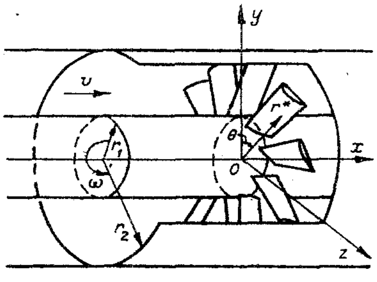


Fig. 1

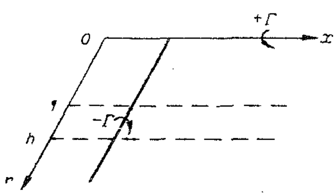


Fig. 2

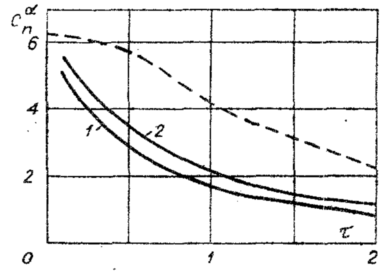


Fig. 3

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 2, pp. 119-127, March-April, 1979. Original article submitted March 20, 1978.

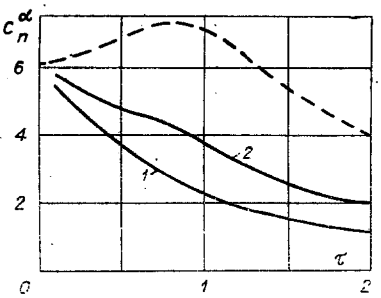


Fig. 4

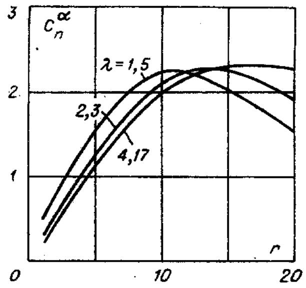


Fig. 5

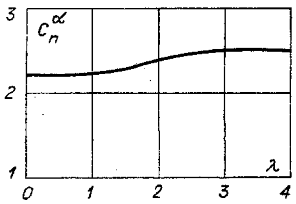


Fig. 6

The screw surfaces which schematically represent the blades and the vortex trails behind them are defined by the equations

$$x = r_1 m\theta, \quad y = r_1 r \cos(\theta + \alpha_n), \quad z = r_1 r \sin(\theta + \alpha_n), \tag{1.1}$$

where $m = v/\omega r_1$; $r = r^*/r_1$ are the dimensionless radial coordinate, and the angle $\theta = \theta^* - \alpha_n$.

The perturbed motion of the liquid outside the blades and the vortex trails with the above assumptions will be potential and steady [10].

The potential of the velocity $\varphi$ of this motion satisfies Laplace's equation

$$\Delta\varphi = 0$$

outside the surfaces $\Sigma_n$ and $W_n$ $(n = 0, \ldots, N-1)$ and the following boundary conditions: nonpenetration of the liquid through the surface of the blades

$$(\mathbf{v}\cdot\nabla)\varphi = V_\nu, \quad \mathbf{x} \in \Sigma_n; \tag{1.2}$$

continuity of the pressure and the normal component of the velocity for a transition through the surface of a vortex trail

$$[p] = 0, \quad [(\mathbf{v}\cdot\nabla)\varphi] = 0, \quad \mathbf{x} \in W_n;$$

decay of the perturbed velocities at infinity in front of the blade

$$\lim_{x\to-\infty} \nabla\varphi = 0;$$

finiteness of the velocity of the liquid along the lines of the front edges of the blades $L_n$

$$\nabla\varphi\,|_{L_n} < \infty;$$

nonpenetration of the liquid through the surface of the outer and inner cylinders

$$\partial\varphi/\partial r = 0 \quad \text{for } r = 1 \text{ and } r = h, \tag{1.3}$$

where $\boldsymbol{\nu}$ is the vector of the normal to the surface $\Sigma_n$; $V_\nu$ is the normal component of the projection of a blade on its mean surface $\Sigma_n$; $\nabla \equiv \text{grad}$; $\mathbf{x} = (x, y, z)$; $\Delta$ is Laplace's operator; $h = r_2/r_1$; and the brackets denote sudden changes in the quantity enclosed in them.

2. The problem can be solved using the scheme in which the blade is replaced by a vortex surface, when the parameter $h \gg 1$ and $h \sim 1$.

We will first consider the case when $h \gg 1$. We will divide the blade into $N_1$ zones with respect to r, $N_2$ zones with respect to $\theta$, and we will simulate it $M = N_1 N_2$ with horseshoe-shaped vortices in the same way as in [13] for a single-plan impeller of finite swing.

The horseshoe-shaped vortex consists of a section of an additional vortex of swing $2\delta r = (h-1)/N_1$ and two semiinfinite vortex filaments converging from the ends of the additional vortex and situated on the screw lines defined by equations (1.1) for fixed values of the coordinate r equal to the coordinates of the ends of the associated vortex. All the vortices of this system have the same intensity $\Gamma_+$, which can be represented in the form

$$\Gamma_+ = v_0 r_1 \Gamma,$$

where $\Gamma$ is a certain dimensionless constant, and $v_0$ is the velocity of the unperturbed flow along the mean radius of the channel $[r = r_0 = (h+1)/2]$.

The choice of the coordinates of the associated vortices and the control points at which the velocity is determined, due to the system of screw horseshoe-shaped vortices, is made in the $(r, \theta)$ plane using the same scheme as in [13].

Suppose s is the number of the zone with respect to $\theta$ $(s = 1, \ldots, N_2)$, k is the number of a zone with respect to r $(k = 1, \ldots, N_1)$, and j is the number of the horseshoe-shaped vortex. We can then introduce the following numbering system:

$$j = N_1(s-1) + k,$$

while the coordinates of the middles of the sections of associated vortices $(r_j, \theta_j)$ and the control points $(r_{0j}, \theta_{0j})$ are given by the equations

$$\begin{gathered} r_{0j} = r_j = h + \delta r(1 - 2k), \\ \theta_j = \psi((0.5 + 2(s-1))/N_2 - 1), \\ \theta_{0j} = \psi((1.5 + 2(s-1))/N_2 - 1). \end{gathered} \tag{2.1}$$

Since the vector of the normal to the screw surface at the point $(r_{0j}, \theta_{0j})$ is given by the expression

$$\mathbf{v} = \frac{1}{\sqrt{m^2 + r_{0j}^2}} (r_{0j},\ m \sin \theta_{0j},\ -m \cos \theta_{0j}),$$

carrying out the integration with respect to r for the normal component of the velocity at the point $(r_{0j}, \theta_{0j})$ from the i-th associated vortex we obtain

$$v_{\nu+}^i (r_{0j}, \theta_{0j}) = -\frac{\Gamma_i}{4\pi} a_{ij} \left[ \frac{r_i + \delta r - r_{0j} \cos(\vartheta_{ij} - \alpha_n)}{R(r_i + \delta r)} - \frac{r_i - \delta r - r_{0j} \cos(\vartheta_{ij} - \alpha_n)}{R(r_i - \delta r)} \right], \tag{2.2}$$

where

$$\begin{gathered} a_{ij} = \frac{r_{0j}^2 \sin(\vartheta_{ij} - \alpha_n) + m^2 \vartheta_{ij} \cos(\vartheta_{ij} - \alpha_n)}{\sqrt{m^2 + r_{0j}^2} \left[ m^2 \vartheta_{ij}^2 + r_{0j}^2 \sin^2(\vartheta_{ij} - \alpha_n) \right]}, \\ R^2(r) = m^2 \vartheta_{ij}^2 + r_{0j}^2 + r^2 - 2 r_{0j} r \cos(\vartheta_{ij} - \alpha_n);\ \vartheta_{ij} = \theta_{0j} - \theta_i. \end{gathered} \tag{2.3}$$

Similarly for the normal component of the velocity from the free vortex belonging to the i-th horseshoe-shaped vortex and having a coordinate r along the height of the blade, we obtain

$$v_{\nu-}^i (r_{0j}, \theta_{0j}, r) = -\frac{\Gamma_i}{4\pi \sqrt{m^2 + r_{0j}^2}} \int_{\theta_i}^{\infty} \frac{r_{0j}(r^2 - m^2) + r(m^2 - r_{0j}^2) \cos \Omega + r m^2 (\theta_{0j} - \theta) \sin \Omega}{R_1^3(r, \theta)} d\theta, \tag{2.4}$$

where $\Omega = \theta_{0j} - \theta - \alpha_n$; $R_1^2(r, \theta) = m^2(\theta_{0j} - \theta)^2 + r_{0j}^2 + r^2 - 2 r_{0j} r \cos \Omega$. Then the normal component of the velocity from the i-th horseshoe-shaped vortex at the point $(r_{0j}, \theta_{0j})$ is

$$v_\nu^i (r_{0j}, \theta_{0j}) = v_{\nu+}^i (r_{0j}, \theta_{0j}) + v_{\nu-}^i (r_{0j}, \theta_{0j}, r_i + \delta r) - v_{\nu-}^i (r_{0j}, \theta_{0j}, r_i - \delta r) = w_{ji}^n \Gamma_i. \tag{2.5}$$

Since the conditions for flow around all the blades are the same, the total velocity from the vortex system introduced at the point $(r_{0j}, \theta_{0j})$ of the zeroth blade $(n = 0)$ will be

$$v_\nu (r_{0j}, \theta_{0j}) = \sum_{i=1}^{M} \Gamma_i \sum_{n=0}^{N-1} w_{ji}^n.$$

The vortex system which replaces the blades of the ring array induces normal velocities on the surfaces of the cylinders. Hence, to satisfy the boundary conditions (1.3), following the idea suggested in [14], we will introduce an additional vortex system which is the reflection with respect to the circles $r = 1$ and $r = h$ of the vortex system of the blades at each cross section $x = \text{const}$. In this case the normal velocities on the surfaces of the cylinders induced by the reflected vortex system will not completely compensate the normal velocities occurring from the vortex system replacing the blades of the array. However, to a first approximation we can assume that the boundary conditions will be satisfied over the whole surface of the cylinders. This problem was investigated in [14] when investigating the flow around a wing with a cylindrical fuselage, and it was concluded that calculations using the reflected system are fairly accurate. Calculations carried out by the author for the case of coaxial cylinders enabled a similar conclusion to be drawn.

The normal velocities induced by the reflected vortex system are found from Eqs. (2.2)-(2.5) in which we must replace $r_i \pm \delta r$ by $1/(r_i \pm \delta r)$ or $h^2/(r_i \pm \delta r)$, respectively.

The total normal velocity induced by all the vortex systems will be given by

$$v_{v1}(r_{0j}, \theta_{0j}) = \sum_{i=1}^{M} \Gamma_i \sum_{n=0}^{N-1} (w_{ji}^n - w_{1ji}^n - w_{2ji}^n), \tag{2.6}$$

where the quantities $w_{1ji}^n$ and $w_{2ji}^n$ define the normal velocities induced by the vortex systems reflected with respect to the cylinders $r = 1$ and $r = h$, respectively.

Substituting Eq. (2.6) into the boundary condition (1.2) we obtain a system of algebraic equations for determining the intensity of the associated vortices

$$AX = B, \tag{2.7}$$

where $A = \{A_{ji}\}$ is a matrix, the elements of which are given by the equation

$$A_{ji} = \sum_{n=0}^{N-1} (w_{ji}^n - w_{1ji}^n - w_{2ji}^n);$$

$X = \{\Gamma_i\}$ is a vector composed of the unknown intensities of the associated vortices, and $B = \{V_\nu(r_{0j}, \theta_{0j})\}$.

Solving the system of equations (2.7) we obtain $\Gamma_i$ $(i = 1, \ldots, M)$, after which we can calculate all the aerodynamic characteristics of both the blades as a whole and their cross sections. To do this it is necessary to use Zhukovskii's theorem [13] for calculating the pressure drop

$$[p] = -\rho v_1(r)\gamma(r, \theta),$$

where $\gamma(r, \theta)$ is the intensity of the associated vortices continuously distributed over the surface of a blade, $\rho$ is the density of the liquid, and $v_1 = \sqrt{v^2 + \omega^2 r_1^2 r^2}$ is the velocity of the unperturbed flow of liquid with respect to the blades.

Denoting by dS the element of area of a blade and using Eq. (1.1) we obtain

$$dS = r_1^2 \sqrt{m^2 + r^2} dr d\theta.$$

Then the aerodynamic force acting on a blade is

$$P = -\rho r_1^2 \int_1^h \int_{-\psi}^{\psi} v_1(r) \sqrt{m^2 + r^2} \gamma(r, \theta)\, dr d\theta \tag{2.8}$$

or, in dimensionless form, $C_n = P \Big/ \frac{1}{2} \rho v_0^2 S$.

Now replacing $\gamma\sqrt{m^2 + r_i^2} d\theta$ by $v_0 \Gamma_i$ and dr by $2\delta r$ in Eq. (2.8) and changing from integrals to finite sums we obtain

$$C_n = -\frac{4\delta r}{S_1 \sqrt{m^2 + r_0^2}} \sum_{i=1}^{M} \sqrt{m^2 + r_i^2} \Gamma_i.$$

For the coefficients of the aerodynamic forces acting on the k-th cross section with respect to the height of the blade, $C_{nk} = P_k \Big/ \frac{1}{2} \rho v_1(r_k) S_k$, we have

$$C_{nk} = -\frac{4\delta r \sqrt{m^2 + r_0^2}}{S_{1k}(m^2 + r_k^2)} \sum_{i=1}^{N_1} \sqrt{m^2 + r_i^2} \Gamma_{N_1 i + k},$$

where $S_1 = S/r_1^2$ and $S_{1k} = S_k/r_1^2$ are the dimensionless areas of a blade and its k-th zone with respect to the height, respectively.

The algorithm for calculating the improper integals in Eq. (2.4) has a considerable influence on the calculation accuracy. We will introduce the variable $x = \theta - \theta_{0j}$ and divide the range of integration into two ranges: $[-\vartheta_{ij}, \Delta]$ and $[\Delta, \infty)$, where $\Delta \gg 1$. The integral over the second range is evaluated using the asymptotic expansion of the function under the integral for $x \gg 1$ and subsequent integration by parts. To evaluate the integral from $-\vartheta_{ij}$ to $\Delta$ we add and subtract the expression

$$\int_{-\vartheta_{ij}}^{\Delta} \frac{r_{0j}(r^2 - m^2) + r(m^2 - r_{0j}^2)}{[(m^2 + rr_{0j})x^2 + (r_{0j} - r)^2]^{3/2}} dx,$$

thereby eliminating the singularity of the function under the integral when the control point $(r_{0j}, \theta_{0j})$ is situated in the region of a horseshoe-shaped vortex. A comparison of the results of the evaluation using this algorithm with the accurate values of the integrals for $\vartheta_{ij} = 0$ and $r_{0j} = 0$ gives a difference in only the third decimal place.

When the parameter h is close to 1 the aerodynamic characteristics vary only slightly along the height of the ring channel. Hence, the intensity of the associated vortices by which the blade is replaced can be assumed to be constant in the radial direction. Hence it follows that there will be no free vortices in the flow behind the blade when $1 < r < h$. At the ends of the blades ($r = 1$ and h) free vortex filaments are not formed due to the boundary condition (1.3). Hence, the associated vortices must continually extend in the radial direction through the boundary of the region of flow up to the x axis and to infinity.

We will divide the blade into $N_2$ zones with respect to $\theta$ and replace each of them by a rectilinear semi-infinite vortex from the axis directed along r. Since the vortices cannot end in the liquid it is necessary to introduce a system of free axial vortices emerging from the ends of the associated vortices (Fig. 2). However, each vortex $\Gamma_i$ of the system introduced induces radial velocities on the surfaces of the cylinders which are given by the equation

$$v_r^i = \frac{\Gamma_i}{4\pi} \sum_{k=0}^{N-1} \frac{\xi \sin(\eta + \alpha_k)}{\xi^2 + r^2 \sin^2(\eta + \alpha_k)} \left[ 1 + \frac{r \cos(\eta + \alpha_k)}{(\xi^2 + r^2)^{1/2}} \right], \tag{2.9}$$

where $\xi = m\theta_i - x$; $\eta = \theta_i - \theta$; $\theta_i$ is the coordinate of the i-th vortex defined by Eqs. (2.1), and $r = 1$ or h.

Since the function $v_r^i$ is odd with respect to $\eta$ with period $2\pi/N$, we can determine the coefficients of the Fourier series $a_n(\xi)$ and $b_n(\xi)$ of this function for $r = h$ and 1, respectively. As can be seen from Eq. (2.9), the coefficients $a_n(\xi)$ and $b_n(\xi)$ are odd functions of $\xi$ and, consequently, the radial velocity on the outer cylinder can be represented in the form

$$v_r^i(\xi, h, \eta) = \frac{2}{\pi} \sum_{n=1}^{\infty} \sin nN\eta \int_0^\infty \sin \tau\xi d\tau \int_0^\infty a_n(t) \sin \tau t dt.$$

A similar representation also occurs for $v_r^i(\xi, 1, \eta)$ with $a_n$ replaced by $b_n$.

In order to satisfy the boundary condition (1.3) we must introduce the additional potential $\varphi_+$, which is given by the expression

$$\varphi_+ = \frac{2}{\pi} \sum_{n=1}^{\infty} \sin nN\eta \left\{ \int_0^\infty \mu_s(\tau, r) \frac{\sin \tau\xi}{\tau} d\tau \int_0^\infty a_n(t) \sin \tau t dt + \int_0^\infty \sigma_s(\tau, r) \frac{\sin \tau\xi}{\tau} d\tau \int_0^\infty b_n(t) \sin \tau t dt \right\},$$

where

$$\mu_s = [K_s'(\tau) I_s(\tau r) - I_s'(\tau) K_s(\tau r)] / T_s(\tau);$$
$$\sigma_s = [K_s(\tau r) I_s'(\tau h) - K_s'(\tau h) I_s(\tau r)] / T_s(\tau);$$
$$T_s = I_s'(\tau h) K_s'(\tau) - I_s'(\tau) K_s'(\tau h).$$

Here $s = nN$, and $I_s$ and $K_s$ are modified Bessel functions of the first and second kind, respectively; the primes denote differentiation with respect to the argument.

Requiring that the condition for nonpenetration of the blades (1.2) should be satisfied at the points $(r_0, \theta_{0j})$ $(j = 1, \ldots, N_2)$, we obtain a system of algebraic equations for determining the unknown intensities of the vortices

$$\sum_{i=1}^{N_2} \Gamma_i (w_{ji} - w_{0ji}) = 1,$$

where

$$w_{ji} = -\frac{1}{4\pi\sqrt{m^2 + r_0^2}} \sum_{k=0}^{N-1} \frac{r_0^2 \sin(\vartheta_{ij} - \alpha_k) + m^2\vartheta_{ij} \cos(\vartheta_{ij} - \alpha_k)}{m^2\vartheta_{ij}^2 + r_0^2 \sin^2(\vartheta_{ij} - \alpha_k)} \left[ 1 + \frac{r_0 \cos(\vartheta_{ij} - \alpha_k)}{(m^2\vartheta_{ij}^2 + r_0^2)^{1/2}} \right]$$

are the normal velocities induced by the vortices which replace the blades of the ring array, and

$$w_{0ji} = \frac{1}{\sqrt{m^2 + r_0^2}} \left( \frac{m}{r_0} \frac{\partial \varphi_+}{\partial \eta} - r_0 \frac{\partial \varphi_+}{\partial \xi} \right).$$

The dimensionless coefficient of the total aerodynamic force acting on a blade in this case is given by the expression

$$C_n = -\frac{4\delta r}{S_1}\sum_{i=1}^{N_2}\Gamma_i.$$

3. Using the above algorithms on the BÉSM-6 computer we determined the distributed and total aerodynamic characteristics of a number of ring arrays.

The parameters representing the geometry of an array and the blade were determined at the mean radius of the ring channel $m = r_0 \cot \beta$ ($\beta$ is the angle of stagger of the array), the thickness of the array

$$\tau = \frac{\psi\sqrt{r_0^2 + m^2}}{\pi r_0} N,$$

and the length

$$\lambda = \frac{(h-1)^2}{S_1}. \tag{3.1}$$

The right side of Eq. (2.7) was taken in the form

$$V_\nu(r_{0j}, \theta_{0j}) = \frac{\alpha(r_{0j})\, v_1(r_{0j})}{\alpha(r_0)\, v_0},$$

where $\alpha(r_{0j})$ is the angle of attack in the section $r = r_{0j}$. In the calculations for $h \gg 1$ the change in the angle of attack along the height of the blade was taken to be linear.

Figures 3 and 4 show the coefficient $C_n^\alpha = C_n/\alpha(r_0)$ of the total aerodynamic force acting on a blade in the ring array as a function of the thickness of the array $\tau$ for angles of stagger $\beta = 30°$ and 60°, respectively. The thickness of the array was varied either using the parameter $\psi$ for fixed h and N (N = 4) or by changing the number of blades (N = 4, 8, 12, and 16) for fixed h and $\psi$ ($\psi = 0.34$ for $\beta = 60°$ and $\psi = 0.19635$ for $\beta = 30°$). For each thickness of the blade we had lengths given by Eq. (3.1). The angle of attack varied from 0.15 for r = 1 to 0.05 for r = h. The results of the calculations in both cases agree completely.

Curves 1 correspond to calculations using the algorithm for $h \gg 1$ (h = 20) while curves 2 correspond to calculations using the algorithm for $h \sim 1$ (h = 2). Comparison with the results obtained in the plane theory for a mean radius of the ring channel (the dashed lines) show considerable differences between the results for the mean and greater thicknesses. For small $\tau$ the results differ only slightly, but somewhat less than for the plane theory, which is due to the effect of the extension of the blades.

We can explain this fact as follows. When the blades of the array rotate the liquid in the ring channel is set into rotational motion, which can be simulated by an axial vortex. This motion is preserved later when the liquid emerges from the blade turbine, influencing the value of the load on the blades. Since the energy expended on forming this motion is lost energy, a reduction in the aerodynamic force coefficient $C_n^\alpha$ occurs. As a result of this the motion controlled by the axial vortex is formed to a large extent only for fairly large thicknesses, and when $\tau \ll 1$ its effect is small. The additional reduction in the coefficient $C_n^\alpha$ for calculations using the algorithm for $h \gg 1$ is due to losses of energy in forming vortex trails behind the blades due to the variability of the load in the radial direction. It should also be noted that a change in the parameter h for fixed thickness and angle of stagger of the array has only a small effect on the value of $C_n^\alpha$.

Figure 5 shows the coefficient $C_n^\alpha$ as a function of the height of a blade r for different extensions for an array of thickness $\tau = 1$ and angle of stagger $\beta = 30°$ for h = 20. The angle of attack varied from $\alpha(1) = 0.05$ to $\alpha(h) = 0.15$. The number of blades N was 4, 8, and 16, and the angle $\psi = 0.3927$, 0.19635, and 0.0982 for $\lambda = 1.5$, 2.3, and 4.17, respectively. The results show that as the extension of the blades is increased the nature of the change in the coefficient of the running aerodynamic force approximates the law of variation of the angle of attack along the height of the blade.

Figure 6 shows the coefficient $C_n^\alpha$ of the total aerodynamic force as a function of the extension of the blade $\lambda$ for an array with thickness $\tau = 1$ and angle of stagger $\beta = 30°$. The point corresponding to $\lambda \to 0$ was obtained by calculation using the algorithm for $h \sim 1$ (h = 2); for $\lambda > 0$ the calculations were made using the algorithm for $h \gg 1$ (h = 20). The results of calculations on this example, shown in Fig. 5, were used to construct this relationship. As can be seen from the results, calculations carried out using both algorithms agree quite well in the region of small extensions. The weak dependence of the total aerodynamic force coefficient on the extension of the blade should also be noted.

## LITERATURE CITED


1. G. Yu. Stepanov, Hydrodynamics of Turbine Arrays [in Russian], Fizmatgiz, Moscow (1962).
2. M. I. Zhukovskii, Aerodynamic Calculation of the Flow in Axial Turbines [in Russian], Mashinostroenie, Moscow (1967).
3. Ya. A. Sirotkin, Aerodynamic Design of the Blades of Axial Turbines [in Russian], Mashinostroenie, Moscow (1972).
4. L. A. Simonov, "Spatial flow in hydromachines," Prikl. Mat. Mekh., 5, No. 3, 471-488 (1941).
5. M. Ribaut, "Three-dimensional calculation of flow in turbomachines with the aid of singularities," Trans. ASME, Ser. A, J. Eng. Power, No. 3, 50-58 (1968).
6. G. S. Samoilovich, Nonstationary Flow and Aeroelastic Vibrations of the Arrays of Turbomachines [in Russian], Nauka, Moscow (1971).
7. D. N. Gorelov, V. B. Kurzin, and V. É. Saren, Aerodynamic Arrays in Nonstationary Flow [in Russian], Nauka, Novosibirsk (1971).
8. A. F. de O. Falcao, "Three-dimensional potential flow through a rectilinear cascade of blades," Ing.-Arch., 44, No. 1, 27-41 (1975).
9. G. M. Morgunov, "Three-dimensional flow around the blade systems of turbomachines by a steady-state flow of ideal liquid," Izv. Akad. Nauk SSSR Mekh. Zhidk. Gaza. No. 6, 3-12 (1975).
10. M. Namba, "Lifting surface theory for a rotating subsonic or transonic blade row," Brit. Aeron. Res. Council, Rept. and Memo, No. 3740, 3-75 (1972).
11. P. Salaün, "Pressions aerodynamiques instationnaires sur une grille annulaire en ecoulement subsonique," ONERA, No. 158 (1974).
12. V. P. Vetchinkin and N. P. Polyakhov, Theory and Design of an Air Propeller [in Russian], Oborongiz, Moscow (1940).
13. S. M. Belotserkovskii, A Thin Carrying Surface in a Subsonic Flow of Gas [in Russian], Nauka, Moscow (1965).
14. V. F. Lebedev, "An approximate method of calculating the aerodynamic load distribution on the wing and fuselage at subsonic speeds," Tr. TsAGI, No. 719, 3-75 (1958).


# SOME FORMULATIONS OF BOUNDARY-VALUE PROBLEMS OF L-PLASTICITY


A. F. Revuzhenko and E. I. Shemyakin



UDC 539.379


1. The construction of mathematical models of a deformable medium is usually reduced to describing the relations between the stress and strain (velocity) tensors. Such an approach is based on two hypotheses: 1) The medium is assumed continuous; 2) in constructing the model any infinitesimal volume of the medium is allotted the properties of a macrospecimen if the latter is deformed under certain boundary conditions allowing a homogeneous stress distribution. Thus, if the specimen is deformed elastically, then it is assumed that each volume element is also deformed elastically. This assumption permits the description of the elasticity to reduce to the description of the elastic behavior of the volume element. By analogy, the legitimacy of such a transfer is also assumed in an investigation of the plastic behavior. Hence, as in the theory of elasticity, the problem of constructing a plastic model reduces to describing the plastic behavior of a volume element of a continuous medium.

However, a class of materials can be mentioned for which the hypothesis of identity between the properties of the specimen and its volume element is not satisfied even approximately. Indeed, let a certain specimen disclose plastic properties under definite loads. A situation is possible when the specimen is divided up into discrete slip surfaces on separate parts (blocks) under the loads mentioned. In this case the plastic properties of the specimen are entirely related to not only the inelastic strains of the blocks but also to their relative slips. If the blocks are deformed elastically, then the plastic properties of the specimen depend only on their relative slips.


Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 2, pp. 128-137, March-April, 1979. Original article submitted May 8, 1978.